

$$\begin{aligned} \max \underline{c}'x \quad \eta \quad \max \underline{c}'x \\ Ax = \underline{b} \quad \underline{p}_1 x_1 + \underline{p}_2 x_2 + \dots + \underline{p}_n x_n = \underline{b} \\ \underline{x} \geq \underline{0}, \underline{b} \geq \underline{0} \quad \underline{x}_i \geq 0, \underline{b} \geq 0 \\ A = (\underline{p}_1, \dots, \underline{p}_n) \\ \underline{x}_{i0} = (x_{10}, x_{20}, \dots, x_{m0}, 0, \dots, 0)' \quad x_{i0} > 0 \\ x_{10}c_1 + x_{20}c_2 + \dots + x_{m0}c_m = z_0 \\ x_{ij}\underline{p}_1 + x_{2j}\underline{p}_2 + \dots + x_{mj}\underline{p}_m = \underline{p}_j \\ x_{ij}c_1 + x_{2j}c_2 + \dots + x_{mj}c_m = z_j \end{aligned}$$

Αν $z_j - c_j < 0$ για ένα τουλάχιστον j ($j=1, \dots, n$) τότε η μη ευφύλισημένη β.ε.λ. x_e δεν είναι άριστη.

$$\underline{c}'x_1 \geq \underline{c}'x_0$$

$$x_{10}\underline{p}_1 + x_{20}\underline{p}_2 + \dots + x_{m0}\underline{p}_m = \underline{b} \quad (1)$$

$$x_{10}c_1 + x_{20}c_2 + \dots + x_{m0}c_m = z_0 \quad (2)$$

$\theta \in \mathbb{R}$

$$(1) - \theta(3) \quad (2) - \theta(4)$$

$$(x_{10} - \theta x_{1j})\underline{p}_1 + \dots + (x_{m0} - \theta x_{mj})\underline{p}_m = \underline{b} - \theta \underline{p}_j$$

$$(x_{10} - \theta x_{1j})c_1 + \dots + (x_{m0} - \theta x_{mj})c_m = z_0 - \theta z_j$$

$$\eta \quad (x_{10} - \theta x_{1j})\underline{p}_1 + \dots + (x_{m0} - \theta x_{mj})\underline{p}_m + \theta \underline{p}_j = \underline{b}$$

$$(x_{10} - \theta x_{1j})c_1 + \dots + (x_{m0} - \theta x_{mj})c_m + \theta c_j = z_0 - \theta(z_j - c_j)$$

$$x(\theta) = (x_{10} - \theta x_{1j}, x_{20} - \theta x_{2j}, \dots, x_{m0} - \theta x_{mj}, 0, \dots, \theta, 0, \dots, 0)'$$

$$\theta \geq 0, \quad x_{i0} - \theta x_{ij} \geq 0$$

\downarrow
j-θέση

i) $x_{ij} \leq 0, \forall i, \theta > 0$ μη φραχμένο πρόβλημα.

ii) $x_{ij} > 0$, για τουλάχιστον ένα i .

$$0 < \theta \leq \theta_0, \quad \theta_0 = \min \left\{ \frac{x_{i0}}{x_{ij}} \mid x_{ij} > 0 \right\}, \quad x_{i0} - \theta x_{ij} \geq 0$$

$$x_{ij} > 0, \quad \theta_0 = \frac{x_{i0}}{x_{ij}}$$

$$\underline{X}_1 = x(\theta_0) = (0, \underbrace{x_{20} - \theta x_{2j}}_{P_2}, \dots, \underbrace{x_{m0} - \theta x_{mj}}_{P_m}, 0, \dots, \theta_0, \dots, 0)$$

$$\lambda_2 P_2 + \lambda_3 P_3 + \dots + \lambda_m P_m + \lambda_j P_j = 0, \lambda_5 \neq 0.$$

$$x_{1j} P_1 + \dots + x_{mj} P_m = P_j$$

$$\lambda_j x_{1j} P_1 + (\lambda_2 + \lambda_j x_{2j}) P_2 + \dots + (\lambda_m + \lambda_j x_{mj}) P_m = 0$$

$$\lambda_j x_{1j} = 0 \Rightarrow x_{1j} = 0$$

ΠΑΡΑΔΕΙΓΜΑ:

$$\max 300x_1 + 200x_2$$

$$x_1 + 2x_2 \leq 6$$

$$2x_1 + x_2 \leq 8$$

$$-x_1 + x_2 \leq 1$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

$$\max 300x_1 + 200x_2$$

$$x_1 + 2x_2 + x_3 = 6$$

$$2x_1 + x_2 + x_4 = 8$$

$$-x_1 + x_2 + x_5 = 1$$

$$x_6 + x_6 = 2$$

$$x_i \geq 0, i=1, 2, \dots, 6.$$

μοναδιαίος πίνακας

B	C _B	b	300	200	0	0	0	0	θ
			P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	
P ₃	0	6	1	2	1	0	0	0	6/1 Γ ₁
← P ₄	0	8	2	1	0	1	0	0	8/2 Γ ₂
P ₅	0	1	-1	1	0	0	1	0	Γ ₃
P ₆	0	2	0	1	0	0	0	-1	Γ ₄
Z		0	-300	-200	0	0	0	0	Γ ₅

$$0 \cdot 1 + 0 \cdot 2 + 0 \cdot (-1) + 0 \cdot 0 = -300$$

$$0 \cdot 6 + 0 \cdot 8 + 0 \cdot 1 + 0 \cdot 2$$

B	C _B	b	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	θ	
P ₃	0	2	0	3/2	1	-1/2	0	0	4/3	Γ ₁ ' = Γ ₁ - 1Γ ₂ '
P ₁	300	4	1	1/2	0	1/2	0	0	8	Γ ₂ ' = 1/2 Γ ₂
P ₅	0	5	0	3/2	0	1/2	1	0	10/3	Γ ₃ ' = Γ ₃ - (-1)Γ ₂ '
P ₆	0	2	0	1	0	0	0	1	2	Γ ₄ ' = Γ ₄ - 0Γ ₂ '
Z	1200		0	-50	0	150	0	0		Γ ₅ ' = Γ ₅ + 300Γ ₁ '

B	C _B	b	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	
P ₂	200	4/3	0	1	2/3	-1/3	0	0	Γ ₁ '' = 2/3 Γ ₁ '
P ₁	300	10/3	1	0	-1/3	2/3	0	0	Γ ₂ '' = Γ ₂ ' - 1/2 Γ ₁ ''
P ₅	0	3	0	0	-1	1	1	0	Γ ₃ '' = Γ ₃ ' - 3/2 Γ ₁ ''
P ₆	0	2/3	0	0	-2/3	1/3	0	1	Γ ₄ '' = Γ ₄ ' - 1 · Γ ₁ ''
Z	3800/3		0	0	1000/3	400/3	0	0	Γ ₅ '' = Γ ₅ ' + 50Γ ₁ ''

Apa $x_1 = \frac{10}{3}$, $x_2 = \frac{4}{3}$, $Z = \frac{3800}{3}$

$(x_1, x_2, x_3, x_4, x_5, x_6)' = (10/3, 4/3, 0, 0, 0, 0)'$

$Z(x(\theta)) = z_0 - \theta(z_j - c_j)$

$x(\theta) = (x_{10} - \theta x_{1j}, x_{20} - \theta x_{2j}, \dots, x_{m0} - \theta x_{mj}, 0, \dots, \theta, \dots, 0)'$

$\theta < \min \left\{ \frac{x_{10}}{x_{1j}}, x_{ij} > 0 \right\}$

$B = (P_1, P_2, \dots, P_m)$

$x_{10}P_1 + x_{20}P_2 + \dots + x_{m0}P_m = b$

$P_j = x_{1j}P_1 + \dots + x_{mj}P_m$

$Z_j = c_1 x_{1j} + \dots + c_m x_{mj}$

$B = (P_3, P_4, P_5, P_6)$

$P_1 = 1 \cdot P_3 + 2P_4 + (-1)P_5 + 0P_6$

$$z_1 = 1 \cdot c_3^{z_0} + 2c_4^{z_0} + (-1)c_5^{z_0} + 0 \cdot c_6^{z_0} = 0.$$

$$z_1 - c_1 = -300.$$

$$B y_j = \underline{r_j}$$

$$y_j = B^{-1} \underline{r_j}$$

$$z_j = \underline{c_B}' B^{-1} \underline{r_j}$$

$$B x_B = \underline{b}$$

$$x_B = B^{-1} \underline{b}$$

$$z_0 = \underline{c_B}' B^{-1} \underline{b}$$

$$y = B^{-1} A$$

$B^{-1} \underline{b}$	$B^{-1} A$
$\underline{c_B}' B^{-1} \underline{b}$	$\underline{c_B}' B^{-1} A$

ΠΑΡΑΔΕΙΓΜΑ:

$$\max -x_1 + 2x_2 - 3x_3$$

$$x_1 - x_2 + x_3 + 2x_4 = 10$$

$$2x_2 - x_3 \leq 1$$

$$x_2 + 2x_4 \leq 8$$

$$x_i \geq 0.$$

$$\max(-x_1 + 2x_2 - 3x_3)$$

$$x_1 - x_2 + x_3 + 2x_4 = 10$$

$$2x_2 - x_3 + x_5 = 1$$

$$x_2 + 2x_4 + x_6 = 8$$

$$x_i \geq 0, i=1, \dots, 6.$$

			-1	2	-3	0	0	0	0	
B	$\underline{c_B}$	\underline{b}	P_1	P_2	P_3	P_4	P_5	P_6	0	
P_1	-1	10	1	-1	1	2	0	0	10/2	Γ_1
P_5	0	1	0	2	-1	0	1	0	1/2	Γ_2
P_6	0	8	0	1	0	2	0	1	8/2	Γ_3
Z	-10	0	0	-1	2	-2	0	0		Γ_4

B	$\underline{c_B}$	\underline{b}	P_1	P_2	P_3	P_4	P_5	P_6	0	
P_1	0	2	1	-2	1	0	0	-1		$\Gamma_1' = \Gamma_1 - 2\Gamma_3'$
P_5	0	1	0	2	-1	0	1	0		$\Gamma_2' = \Gamma_2 - 2\Gamma_3'$
P_4	2	4	0	1/2	0	1	0	1/2		$\Gamma_3' = \Gamma_3/2$
Z	-2	0	0	0	2	0	0	1		$\Gamma_4' = \Gamma_4 + 2\Gamma_3'$

⊕ έχω και ένα -2 στην θέση

$$x_1=2, x_2=0, x_3=0, x_4=4, x_5=1, x_6=0, z=-2$$

B	CB	b	P_1	P_2	P_3	P_4	P_5	P_6
P_1	-1	3	1	0	0	0	1	-1
P_2	2	1/2	0	1	-1/2	0	1/2	0
P_4	0	15/4	0	0	1/4	1	-1/4	1/2
Z	-2	0	0	0	2	0	0	1

$$x_1=3, x_2=1/2, x_3=0, x_4=15/4, x_5=0, x_6=0$$

$$x' = \lambda x_1' + (1-\lambda)x_2'$$

$$= \lambda(2, 0, 0, 4, 1, 0)' + (1-\lambda)(3, 1/2, 0, 15/4, 0, 0)'$$

$$0 \leq \lambda \leq 1$$

ΠΑΡΑΔΕΙΓΜΑ:

$\min (x_1 + x_2 - x_3)$	$-\max (-x_1 - x_2 + x_3)$
$2x_1 + 2x_2 - x_3 \leq 2$	$2x_1 + 2x_2 - x_3 + x_4 = 2$
$-x_1 - 7x_2 + 2x_3 \leq 2$	$-x_1 - 7x_2 + 2x_3 + x_5 = 2$
$7x_1 + x_2 - x_3 \leq 10$	$7x_1 + x_2 - x_3 + x_6 = 10$
$4x_1 + 6x_2 - 2x_3 \leq 6$	$4x_1 + 6x_2 - 2x_3 + x_7 = 6$
$x_1, x_2, x_3 \geq 0$	$x_i \geq 0, i=1, \dots, 7$

B	CB	b	P_1	P_2	P_3	P_4	P_5	P_6	P_7
P_4	0	2	2	2	-1	1	0	0	0
P_5	0	2	-1	-7	2	0	1	0	0
P_6	0	10	7	1	-1	0	0	1	0
P_7	0	6	4	6	-2	0	0	0	1
Z	0	1	1	1	-1	0	0	0	0

→ όλα ασήμια, όλα μη φραγμένο πρόβλημα

P_4	0	3	3/2	-3/2	0	1	1/2	0	0	$\Gamma_1' = \Gamma_1 - (-1)\Gamma_2'$
P_5	1	1	-1/2	-7/2	1	0	1/2	0	0	$\Gamma_2' = \Gamma_2/2$
P_6	0	11	13/2	-5/2	0	0	1/2	1	0	$\Gamma_3' = \Gamma_3 - (-1)\Gamma_2'$
P_7	0	8	13/2	-1	0	0	1	0	1	$\Gamma_4' = \Gamma_4 - (2)\Gamma_2'$
			4/2	-5/2	0	0	1/2	0	0	$\Gamma_5' = \Gamma_5 - (-1)\Gamma_2'$